

New Parametrization of Neutrino Mixing Matrix

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Abstract

Global fits to neutrino oscillation data are compatible with tri-bimaximal mixing pattern, which predict $\theta_{23} = \frac{\pi}{4}$, $\theta_{12} = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ and $\theta_{13} = 0$. We propose here to parametrize the tri-bimaximal mixing matrix V_{TBM} by its hermitian generator H_{TBM} using the exponential map. Then we use the exponential map to express the deviations from tri-bimaximal pattern by deriving the hermitian matrices $H_{z=0}$ and H_1 . These deviations might come from the symmetry breaking of the neutrino and charged lepton sectors.

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In recent years, masses and mixing of different generations of neutrinos have been the subject of numerous experimental and theoretical studies. These efforts not only confirmed the oscillations of neutrinos but also measured the mass-squared differences of the neutrino mass eigenstates.

Neutrino oscillation was suggested by Pontecorvo [1] fifty years ago by analogy with the Kaon system and further generalized to three flavors by Maki, Nakagawa and Sakata (PMNS) [2]. Visually, these mixings could be regarded as a rotation from fermion mass eigenstates to flavor eigenstates. The phenomenon of neutrino mixing can be simply described by the PMNS unitary matrix V , which links the neutrino flavor eigenstates ν_e, ν_μ, ν_τ to the mass eigenstates ν_1, ν_2, ν_3 :

$$V = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \quad (1)$$

In the standard parametrization used by the Particle Data Group (PDG), the PMNS matrix is expressed by three mixing angles θ_{12}, θ_{23} and θ_{13} and one intrinsic CP violating phase δ for Dirac neutrinos,

$$V(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (2)$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ and for Majorana neutrino, there are additional two independent Majorana CP violating phases given by a diagonal matrix $K = \text{diag}(1, e^{\sigma_1}, e^{\sigma_2})$. We will ignore the Majorana CP violating phase in this work.

The merit of this parametrization is that the three mixing angles are directly related to the mixing angles of solar, atmospheric and CHOOZ reactor neutrino oscillations, namely :

$$\begin{aligned} \theta_{12} &\approx \theta_{\text{sun}} \\ \theta_{23} &\approx \theta_{\text{atm}} \\ \theta_{13} &\approx \theta_{\text{chz}} \end{aligned} \quad (3)$$

Moreover, the measure of CP violation is expressed through the rephasing invariant Jarlskog parameter defined as [3] :

$$J = \text{Im} (V_{e2} V_{\mu 3} V_{e3}^* V_{\mu 2}^*) \quad (4)$$

The exact expressions between the sines of the three mixing angles and the moduli of the elements of the PMNS mixing matrix are given by :

$$\begin{aligned} \sin^2 \theta_{13} &= |V_{e3}|^2 \\ \sin^2 \theta_{12} &= \frac{|V_{e2}|^2}{1 - |V_{e3}|^2} \\ \sin^2 \theta_{23} &= \frac{|V_{\mu 3}|^2}{1 - |V_{e3}|^2} \end{aligned} \quad (5)$$

Our current knowledge on the magnitude of the elements of the neutrino mixing matrix comes from the experiments on neutrino oscillations. A recent global 3ν oscillation analysis of neutrino experimental data gives the following numerical ranges [4] :

Parameter	$\sin^2 \theta_{12}$	$\sin^2 \theta_{23}$	$\sin^2 \theta_{13}$
Best fit	0.312	0.466	0.016
1σ range	$0.294 - 0.331$	$0.408 - 0.539$	$0.006 - 0.026$
2σ range	$0.278 - 0.352$	$0.366 - 0.602$	< 0.036
3σ range	$0.263 - 0.375$	$0.331 - 0.644$	< 0.046

TABLE I: Global 3ν oscillation analysis with best fit values and allowed n_σ ranges.

These experimental data are consistent with the so-called tri-bimaximal mixing (TBM) matrix pattern where the low energy neutrino mixing matrix V_{TBM} [5] is given by :

$$\begin{aligned} V_{TBM} &= R_{23}(x_0) R_{12}(y_0) \\ &= \begin{pmatrix} \cos y_0 & \sin y_0 & 0 \\ -\cos x_0 \sin y_0 & \cos x_0 \cos y_0 & \sin x_0 \\ \sin x_0 \sin y_0 & -\sin x_0 \cos y_0 & \cos x_0 \end{pmatrix} \end{aligned} \quad (6)$$

with R_{23} and R_{12} are rotations with angle $x_0 = \pi/4$ and $y_0 = \sin^{-1}(1/\sqrt{3})$.

The TBM mixing has been studied extensively in the literature [6]. Parametrization of

neutrino mixing matrix with TBM as the zeroth order has been considered by several authors with different approaches [7]. Deviations from $\theta_{13} = 0$ due to charged lepton corrections [8] and renormalization group running effects [9] have been considered in literature.

In this paper, we propose to parametrize the TBM mixing matrix V_{TBM} by its generator H_{TBM} using the exponential map. Indeed, in Lie theory of groups and their corresponding algebras the exponential map is a crucial tool because it gives the connection between Lie algebra element and their corresponding Lie group element. Using this connection, we derive the generator $H_{z=0}$ of the mixing matrix $V_{z=0}$. Then we use the exponential map to find the hermitian generator H_1 which expresses the deviations from $z = \theta_{13} = 0$. For us, both hermitian matrices $H_{z=0}$ and H_1 encode deviations from TBM hermitian generator H_{TBM} since there is no convincing reason for TBM to be exact. Moreover, we consider that the $H_{z=0}$ arises from the diagonalization of the neutrino mass matrix M_ν , whereas H_1 comes from the diagonalization of the charged lepton sector M_l .

The unitary TBM matrix has three distinct eigenvalues λ_i^0 as follows :

$$\begin{aligned}\lambda_1^0 &= 1 \\ \lambda_{2,3}^0 &= \frac{\text{Tr}(V_{TBM}) - 1 \pm iN_0}{2}\end{aligned}\tag{7}$$

Here N_0 is a normalizing factor which can be expressed in terms of the trace of TBM mixing matrix as :

$$N_0 = \sqrt{4 - (\text{Tr}(V_{TBM}) - 1)^2}\tag{8}$$

Based on this we write V_{TBM} in terms of its eigenvalues as :

$$V_{TBM} = U^\dagger \Lambda_0 U\tag{9}$$

where $\Lambda_0 = \text{diag}(\lambda_1^0, \lambda_2^0, \lambda_3^0)$ and the diagonalizing unitary matrix U is given by :

$$U = \frac{1}{N_0} \begin{pmatrix} \sin x_0 (1 + \cos y_0) & \sin x_0 \sin y_0 & \sin y_0 (1 + \cos x_0) \\ -\sqrt{2} (\lambda_2^0 + 1) \cos(\frac{y_0}{2}) & \sqrt{2} (\lambda_2^0 - \cos x_0) \cos(\frac{y_0}{2}) & \sqrt{2} \sin x_0 \cos(\frac{y_0}{2}) \\ -\sqrt{2} (\lambda_3^0 + 1) \cos(\frac{y_0}{2}) & \sqrt{2} (\lambda_3^0 - \cos x_0) \cos(\frac{y_0}{2}) & \sqrt{2} \sin x_0 \cos(\frac{y_0}{2}) \end{pmatrix}\tag{10}$$

Now the diagonal matrix $\Lambda_0 = \text{diag}(\lambda_1^0, \lambda_2^0, \lambda_3^0)$ which has three roots of unity can be rewritten as :

$$\Lambda_0 = e^{i \Omega_0} \quad (11)$$

where Ω is given by :

$$\Omega_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \omega_0 & 0 \\ 0 & 0 & -\omega_0 \end{pmatrix} \quad (12)$$

and $\omega_0 = -i \ln(\lambda_2^0) = \arctan\left(\frac{N_0}{Tr(V_{TBM})-1}\right)$.

With this approach, we can express the TBM mixing matrix V_{TBM} using the exponential map as :

$$\begin{aligned} V_{TBM} &= U^\dagger e^{i \Omega_0} U \\ &= e^{i H_{TBM}} \end{aligned} \quad (13)$$

where the Hermitian matrix H_{TBM} is :

$$\begin{aligned} H_{TBM} &= U^\dagger \Omega U \\ &= \frac{i\omega_0}{N_0} \begin{pmatrix} 0 & -(1 + \cos x_0) \sin y_0 & \sin x_0 \sin y_0 \\ (1 + \cos x_0) \sin y_0 & 0 & -\sin x_0 (1 + \cos y_0) \\ -\sin x_0 \sin y_0 & \sin x_0 (1 + \cos y_0) & 0 \end{pmatrix} \end{aligned} \quad (14)$$

As we know, the exponential map connects Lie algebra element to their corresponding Lie group element. Here H_{TBM} is the generator for the TBM mixing matrix.

In this parametrization, the Hermitian matrix H_{TBM} gives the zeroth order of the neutrino mixing matrix and for TBM values $x_0 = \frac{\pi}{4}$ and $y_0 = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$, it has the following simple expression :

$$H_{TBM} = i \frac{\arctan\left(\frac{\sqrt{4 - \left(-1\sqrt{\frac{2}{3}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)^2}}{-1 + \sqrt{\frac{2}{3}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}}\right)}{\sqrt{4 - \left(-1\sqrt{\frac{2}{3}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)^2}} \begin{pmatrix} 0 & -\left(\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{6}}\right) & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{6}} & 0 & -\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} & 0 \end{pmatrix}$$

$$= i \begin{pmatrix} 0 & -0.5831 & 0.2415 \\ 0.5831 & 0 & -0.7599 \\ -0.2415 & 0.7599 & 0 \end{pmatrix} \quad (15)$$

The above parametrization of TBM (14) is written in terms of mixing angles x_0 and y_0 , but it is easy to see that the elements of the Hermitian matrix H_{TBM} are related directly to the elements of the mixing matrix V_{TBM} as :

$$H_{TBM} = \frac{i\omega_0}{N_0} \begin{pmatrix} 0 & V_{TBM\ 21} - V_{TBM\ 12} & V_{TBM\ 31} - V_{TBM\ 13} \\ -V_{TBM\ 21} + V_{TBM\ 12} & 0 & V_{TBM\ 32} - V_{TBM\ 23} \\ -V_{TBM\ 31} + V_{TBM\ 13} & -V_{TBM\ 32} + V_{TBM\ 23} & 0 \end{pmatrix} \quad (16)$$

where ω_0 and N_0 are expressed by the invariant parameter $Tr(V_{TBM}) = V_{TBM\ 11} + V_{TBM\ 22} + V_{TBM\ 33}$.

Many future experiments are geared towards improving the precision measurements of the mixing angles and mass-squared differences, so departure from the exact tri-bimaximal mixing is expected. The signature of the deviation from the TBM mixing scenario comes from the measurement of a non-zero value for V_{e3} and/or the observation of CP violation in neutrino oscillations.

The deviation from TBM mixing scheme can be parametrized by writing the PMNS matrix V in terms of the exponential map $V_{z=0} = e^{iH_{z=0}}$ as :

$$V(x, y, z, \delta) = V_{z=0} V_1 \quad (17)$$

where the Hermitian matrix $H_{z=0}$ has the same form as H_{TBM} ,

$$H_{z=0} = \frac{i\omega}{N} \begin{pmatrix} 0 & -(1 + \cos x) \sin y & \sin x \sin y \\ (1 + \cos x) \sin y & 0 & -\sin x (1 + \cos y) \\ -\sin x \sin y & \sin x (1 + \cos y) & 0 \end{pmatrix} \quad (18)$$

with $\omega_{z=0} = \arctan \left(\frac{N_{z=0}}{Tr(V_{z=0}) - 1} \right)$ and $N_{z=0} = \sqrt{4 - (Tr(V_{z=0}) - 1)^2}$.

Similarly, we can express the elements of $H_{z=0}$ in terms of the observables of the mixing matrix $V_{z=0}$. We emphasize here that the parametrization in terms of the generator is completely general and is not based on the TBM pattern or $z = 0$ (*see Appendix where a*

more general result of the generator of the mixing matrix with no CP is found and written in terms of observables).

Both $V_{z=0}$ and V_1 encode deviations from tri-bimaximal mixing. Using the exponential map, the matrix $V_1 = e^{iH_1}$ generated by the Hermitian matrix H_1 is given by :

$$H_1 = i \begin{pmatrix} 0 & 0 & -e^{-i\delta} z \cos y \\ 0 & 0 & -e^{-i\delta} z \sin y \\ e^{i\delta} z \cos y & e^{i\delta} z \sin y & 0 \end{pmatrix} \quad (19)$$

CP violation is described by the Jarlskog invariant which to third order in z is :

$$J = \frac{1}{4}z \left(1 - \frac{7}{6}z^2 \right) \sin 2x \sin 2y \sin \delta + O(z^4) \quad (20)$$

The best fit values for $x = \theta_{23}$, $y = \theta_{12}$ and $z = \theta_{13}$ of the global analysis (see table above) leads to :

$$H_{z=0} = i \begin{pmatrix} 0 & -0.5642 & 0.2225 \\ 0.5642 & 0 & -0.7288 \\ -0.2225 & 0.7288 & 0 \end{pmatrix} \quad (21)$$

and

$$H_1 = i \begin{pmatrix} 0 & 0 & -0.1052 e^{-i\delta} \\ 0 & 0 & -0.0160 e^{-i\delta} \\ 0.1052 e^{i\delta} & 0.0160 e^{i\delta} & 0 \end{pmatrix} \quad (22)$$

The Jarlskog invariant is :

$$J = 0.02876 \sin \delta \quad (23)$$

We may consider that $V_{z=0}$ which has TBM structure is coming from neutrino sector and V_1 coming from symmetry breaking in the charged lepton sector. The neutrino mass matrix is given as :

$$\begin{aligned} M_\nu &= V_{z=0} \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) V_{z=0}^T \\ &= \begin{pmatrix} m_{\nu_1} + 2\Delta_{\nu_{21}} \sin^2 y & \Delta_{\nu_{21}} \cos x \sin 2y & -\Delta_{\nu_{21}} \sin x \sin 2y \\ \Delta_{\nu_{21}} \cos x \sin 2y & m_{\nu_2} - 2\Delta_{\nu_{21}} \cos^2 x \sin^2 y + 2\Delta_{\nu_{32}} \sin^2 x & \Delta_{\nu_{21}} \sin 2x \sin^2 y + m_{\nu_{23}} \sin 2x \\ -\Delta_{\nu_{21}} \sin x \sin 2y & \Delta_{\nu_{21}} \sin 2x \sin^2 y + m_{\nu_{23}} \sin 2x & m_{\nu_3} - 2\Delta_{\nu_{21}} \sin^2 x \sin^2 y - 2\Delta_{\nu_{32}} \sin^2 x \end{pmatrix} \end{aligned} \quad (24)$$

Here we have defined $m_{\nu_{ij}} = \frac{m_{\nu_i} + m_{\nu_j}}{2}$ and $\Delta_{\nu_{ij}} = \frac{m_{\nu_i} - m_{\nu_j}}{2}$.

To first order in z , the charged lepton mass matrix is :

$$M_l = V_1 \text{ diag}(m_e, m_\mu, m_\tau) V_1^\dagger \quad (25)$$

$$= \begin{pmatrix} m_e & 0 & e^{-i\delta} \Delta_{\tau e} z \cos y \\ 0 & m_\mu & e^{-i\delta} \Delta_{\tau \mu} z \sin y \\ e^{i\delta} \Delta_{\tau e} z \cos y & e^{i\delta} \Delta_{\tau \mu} z \sin y & m_\tau \end{pmatrix} + O(z^2)$$

where we defined $\Delta_{ij} = \frac{m_i - m_j}{2}$ with $i = e, \mu, \tau$.

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Appendix

We present here a general result for the connection of the Lie group element V and Lie algebra element H without CP phase.

The mixing matrix without CP is given by :

$$V_{\delta=0} = \begin{pmatrix} \cos y \cos z & \sin y \cos z & \sin z \\ -\cos x \sin y - \sin x \cos y \sin z & \cos x \cos y - \sin x \sin y \sin z & \sin x \cos z \\ \sin x \sin y - \cos x \cos y \sin z & -\sin x \cos y - \cos x \sin y \sin z & \cos x \cos z \end{pmatrix} \quad (26)$$

The unitary matrix $V_{\delta=0}$ has three distinct eigenvalues λ_i as follows :

$$\lambda_1 = 1$$

$$\lambda_{2,3} = \frac{\text{Tr}(V_{\delta=0}) - 1 \pm iN_{\delta=0}}{2} \quad (27)$$

Here $N_{\delta=0}$ is a normalizing factor which can be expressed in terms of the trace of mixing matrix $V_{\delta=0}$ as :

$$N_{\delta=0} = \sqrt{4 - (\text{Tr}(V_{\delta=0}) - 1)^2} \quad (28)$$

Using the above approach, it is easy to find the matrix $V_{\delta=0}$ in terms of the exponential map as :

$$V_{\delta=0} = e^{i H_{\delta=0}} \quad (29)$$

where the Hermitian generator matrix can be expressed in terms of the elements of the mixing matrix $V_{\delta=0}$ as :

$$H_{\delta=0} = \frac{i\omega_{\delta=0}}{N_{\delta=0}} \begin{pmatrix} 0 & V_{\delta=0,21} - V_{\delta=0,12} & V_{\delta=0,31} - V_{\delta=0,13} \\ -V_{\delta=0,21} + V_{\delta=0,12} & 0 & V_{\delta=0,32} - V_{\delta=0,23} \\ -V_{\delta=0,31} + V_{\delta=0,13} & -V_{\delta=0,32} + V_{\delta=0,23} & 0 \end{pmatrix} \quad (30)$$

where $\omega_{\delta=0}$ and $N_{\delta=0}$ are expressed by the invariant parameter,

$$\text{Tr}(V_{\delta=0}) = V_{\delta=0,11} + V_{\delta=0,22} + V_{\delta=0,33} \quad (31)$$

$V_{\delta=0}$ is given by :

$$V_{\delta=0} = \mathbf{1}_{3 \times 3} + i \frac{\sin \omega_{\delta=0}}{\omega_{\delta=0}} H_{\delta=0} - \frac{1 - \cos \omega_{\delta=0}}{\omega_{\delta=0}^2} H_{\delta=0}^2 \quad (32)$$

By using best fit, we get :

$$H_{\delta=0} = i \begin{pmatrix} 0 & -0.611 & 0.105 \\ 0.611 & 0 & -0.765 \\ -0.105 & 0.765 & 0 \end{pmatrix} \quad (33)$$

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